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RED SUPERGIANTS AND NEUTRINO EMISSION. II.

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ABSTRACT

The variation with stellar mass of the ratio of the numbers of blue and red supergiants is investigated. Statistical data for supergiants in young open clusters and subgroups of associations are collected to supplement a more restricted list in Paper I. Improved methods are used to identify hydrogen-burning supergiants, as well as faint supergiant remnants of binary mass exchange, and to arrange the bright evolved supergiants in order of their masses. Neither of these two operations requires knowledge of stellar distances or luminosities. Relevant published work on stellar evolution, rotation, mass loss, and duplicity is used to predict upper and lower limits on the blue-to-red ratio. It is concluded that (1) the observed paucity of very massive red supergiants (and of carbon stars) confirms and extends the trend observed in Paper I, and thereby supports the idea of neutrino-induced acceleration of the carbon-burning and later phases of evolution; (2) the h and χ Persei association shows the same dependence of the blue-to-red ratio on stellar mass as do other clusters and associations; and (3) a moderate decrease of the blue-to-red ratio is observed with increasing galactocentric distance in the Galaxy (as in M33) and seems to be due to a relative scarcity of extremely young stars in distant galactic regions.

I. INTRODUCTION

In a recent paper (Stothers 1969*a*, hereinafter called Paper I) the observed ratio of the numbers of blue and red supergiants in young stellar groups containing M-type supergiants was compared with a set of predicted ratios based on theoretical stellar lifetimes. The large observed ratio ($\sim$5) in the youngest groups was taken to imply rapid evolution of red supergiants after core helium burning. Neutrino emission due to the hypothetical electron-neutrino interaction in the current-current theory of weak interactions was presumed to be responsible for this (see also Stothers and Chin 1969). More recently, Schild (1970) and Humphreys (1970*c*), using improved statistical data for supergiants, have added support to the main conclusions of Paper I.

Unfortunately, a number of ambiguities and difficulties in interpretation have come to light, despite the precautions taken in Paper I (Chiosi and Summa 1970; Ferrari *et al.* 1970*a*, *b;* Bandyopadhyay 1971; Bisnovatyi-Kogan and Nadezhin 1972; Eggen 1971; Simpson 1971; Stothers 1972*b*). Some of these problems have been resolved by subsequent work (Stothers and Chin 1970; Stothers and Lloyd Evans 1970; Stothers and Leung 1971; Stothers 1970*c*, 1971*a*, 1972*b*), while others will be taken up with new data in the present paper. Persistent problems include (1) the stage of evolution attained by a particular supergiant, (2) axial rotation, (3) mass loss, (4) duplicity, (5) completeness and bias in the observational data, (6) spread of ages in a stellar group, and (7) luminosity differences due to differences in initial chemical composition.

To obtain an unbiased sample of supergiants, the supergiant members of young clusters and subgroups of associations that do *not* contain M-type supergiants will be tabulated as a supplement to the list of supergiants in Paper I (§ II). New theoretical work is next reviewed in order to predict reliable upper and lower limits on the blue-to-red ratio (§ III). The observed blue-to-red ratio is determined as a function of stellar mass and of position in a galaxy, and is then compared with the predicted ratio (§ IV). An assessment of the status of the test for neutrino emission concludes the paper (§ V). Since the astrophysical test involving massive supergiants is currently the best test for the existence of the electron-neutrino interaction (Stothers 1970*a*),[1] it is considered important here to reexamine all aspects of the blue-to-red ratio very carefully.

II. SUPERGIANTS IN YOUNG OPEN CLUSTERS AND SUBGROUPS OF ASSOCIATIONS

Open clusters and nearby associations that do not contain M-type supergiants are listed in table 1. The format of this table is identical to the format used in table 1 of Paper I. Sources of distance modulus, extinction correction, and bolometric correction are also the same as in Paper I. We have adopted B.C. = −2.8 for an O9.5 supergiant and −3.0 for an O9 supergiant. Following our previous paper,[2] criteria for group membership are (1) position in equatorial coordinates with respect to the group center, (2) radial velocity (available except where indicated under "Remarks" in table 1), and (3) proper motion. Proper motions have been used only for II Persei, Cr 107, NGC 6231 (I Scorpii), and NGC 6823. Besides the groups listed in table 1 and in Paper I, other groups are known to contain massive supergiants, but incompleteness of the data in one respect or another warrants their exclusion from our list.

Because Of stars (e.g., Conti and Alschuler 1971) and Wolf-Rayet stars (e.g., Gebbie and Thomas 1968) are sometimes accounted to be supergiants, we note the following members of the stellar groups discussed in this paper and in Paper I: three classical Of stars in the "outer group" of I Persei, one in IC 1805, and three in NGC 6231 (I Scorpii); and one Wolf-Rayet star in Pi 20, two in NGC 6231 (I Scorpii), and one in NGC 6871. The Wolf-Rayet star observed near Cr 121 is probably not a member (Crampton 1971).

III. THEORETICAL RATIO OF THE NUMBERS OF BLUE AND RED SUPERGIANTS

Early theoretical work on the evolution of massive stars has been summarized in Paper I. New work is summarized here, and various criticisms are answered; the blue-to-red ratio is then predicted.

a) *Hydrogen Burning*

A detailed analysis of the H-R diagrams of very young clusters with the help of theoretical isochrones (Stothers 1972*b*) has recently indicated that, contrary to previous ideas (Paper I; Stothers and Chin 1969; Humphreys 1970*c;* Schild 1970), some of the fainter blue supergiants, as well as all of the blue giants, are probably still burning core hydrogen. Hayashi and Cameron (1962) and Stothers and Lloyd Evans (1970) had already suspected this, but it is now at last possible to actually identify hydrogen-burning supergiants in stellar groups of *well-defined age.*

[1] See also the comments by Ruderman (1969*a*, *b*, 1971), by Domogatskii and Nadezhin (1971), and by Stothers and Chiu (1971).

[2] Further data available for the groups containing M supergiants will be summarized here since they will be used in our final selection of supergiants.

Radial velocities: BD+57°258 in NGC 457 (Humphreys 1970*a*); BD+59°274 (Zug 4) in NGC 581 (Humphreys 1970*a*); HD 16310, HD 17145, HD 17306, BD+58°373, BD+57°524, and BD+58°445 in the outer group of I Persei (Petrie and Pearce 1962; Humphreys 1970*a;* Gahm and Arkling 1971); and HD 100943, CD−60°3621, and CD−60°3636 in NGC 3766 (Feast, Thackeray, and Wesselink 1957; Humphreys, Strecker, and Ney 1972).

Proper motions: supergiants in I Orionis (Lesh 1968), I Persei (Lavdovskii 1965), and II Sco (Jones 1970).

Membership lists, photometry, and spectral types (Humphreys 1970*b*; Lee 1970; Schild 1970; Eggen 1971; Keenan 1971).

Additional stellar groups containing M supergiants: NGC 663 (Humphreys 1970*b*), NGC 2439 (Bidelman 1950; Humphreys 1970*b*), I Cephei (Simonson 1968), I Aurigae (Stothers 1971*b*), and more distant associations (Stothers 1972*c*).

Important revisions of the distance modulus for h and χ Persei and the inner group of I Persei (Crawford, Glaspey, and Perry 1970; Vogt 1971; Lloyd Evans 1972).

TABLE 1

SUPERGIANTS IN OPEN CLUSTERS AND SUBGROUPS OF ASSOCIATIONS WITHOUT RED-SUPERGIANT MEMBERS

Cluster	$(m-M)_0$	$\langle E_{B-V}\rangle$	Star	Sp.	M_v	M_{bol}	Remarks
NGC 957.....	11.7	0.8	HD 15690	B1.5 Ib	−6.3	− 8.0	
IC 1805......	11.7	0.9	BD+60°493	B0.5 Ia	−6.5	− 8.5	
II Persei.....	8.0	0.3	ζ Per	B1 Ib	−6.2	− 8.0	
NGC 2129....	11.3	0.7	HDE 250290	B3 Ib	−6.2	− 7.5	
Cr 107.......	11.2	0.4	HD 47240	B1 Ib	−6.1	− 7.9	
IC 2581......	12.0	0.4	HD 90706	B2.5 Ib	−6.8	− 8.2	
			HD 90772	A7 Ia	−8.6	− 8.5	
Pi 20........	13.2	1.2	Pi 20-11	O9.5 Ib	−6.2	− 9.0	No radial velocity
			Pi 20-7	B0 Ib	−6.2	− 8.5	No radial velocity
			Pi 20-8	B1 Ia:	−9.1	−10.9	No radial velocity
NGC 6231 (I Scorpii)..	11.5	0.5	HD 152424	O9 Ia	−7.3	−10.3	
			HD 152249	O9 Ib	−6.6	− 9.6	
			HD 152003	O9.5 Ib	−6.5	− 9.3	
			HD 152147	O9.5 Ib	−6.3	− 9.1	
			HD 152405	O9.5 Ib	−5.7	− 8.5	
			HD 152234	B0.5 Ia	−7.4	− 9.4	
			HD 152667	B0.5 Ia	−6.8	− 8.8	
			HD 152235	B1 Ia	−7.4	− 9.2	
			ζ^1 Sco	B1.5 Ia-0	−8.8	−10.5	
NGC 6530....	11.0	0.4	HD 164865	B9 Iab	−5.9	− 6.4	No radial velocity
NGC 6823....	12.1	0.8	BD+23°3745	B0.5 Ib	−6.1	− 8.1	
NGC 6871....	11.5	0.4	HD 190918	O9.5 I+W-R	−5.9	− 8.7	
			HDE 227634	B0 Ib	−5.1	− 7.4	
			HD 190919	B1 Ib	−5.6	− 7.4	
			BD+35°3955	B1 Ib	−5.5	− 7.3	
			V448 Cyg	B1 Ib-II+O	−5.4	− 7.2	
NGC 6910....	11.3	1.0	HD 194279	B1.5 Ia	−8.0	− 9.7	
NGC 6913....	11.5	1.0	HDE 229221	B0: I: pe	−5.8	− 8.1	
			HDE 229227	B0 II	−5.3	− 7.6	
			HDE 229238	B0.5 Ib	−6.1	− 8.1	
			HDE 229239	B1 Iab	−5.8	− 7.6	
NGC 7235....	12.4	0.9	HDE 239886	B9 Iab	−6.2	− 6.7	

NOTES TO TABLE 1

NGC 957 (Hoag and Applequist 1965).

IC 1805 (Ishida 1969).

II Persei (Harris 1956): An H-R diagram is given by Seyfert, Hardie, and Grenchik (1960) and by Lesh (1969), who has also examined proper motions.

NGC 2129 (Hoag and Applequist 1965): HD 40003 (B3 Ib) is too distant to be a member.

Cr 107 (Schmidt-Kaler 1968).

IC 2581 (Lloyd Evans 1969).

Pi 20 (Lyngå 1968): Pi 20-8 (HD 134959) may be the most luminous star known (cf. Stothers and Simon 1968).

NGC 6231, Scorpii (Schild, Hiltner, and Sanduleak 1969): HD 152667 is a binary with an 8-day orbital period, and therefore would not have left the vicinity of the main sequence (Stothers and Lloyd Evans 1970). Braes (1967) has measured proper motions for many stars in the direction of this association. Ten M-type stars (HD catalog) are listed by Braes as lying in the general direction, but they have too faint apparent magnitudes or too large proper motions to be supergiant members.

NGC 6530 (Walker 1957): An H-R diagram is given by Hiltner, Morgan, and Neff (1965). HD 164865 is an outlying object and has a color excess which is three times the cluster average, but the star is probably a member on the basis of its corrected apparent magnitude. Two S-type stars are near the cluster (The, quoted in Blanco 1965).

NGC 6823 (Serkowski 1965): Further data are given by Hoag and Applequist (1965). Erickson (1971) has measured proper motions.

NGC 6871 (Hoag and Applequist 1965): Further data are given by Cohen (1969).

NGC 6910 (Hiltner 1956): Membership lists and a color-magnitude diagram are provided by Hoag *et al.* (1961).

NGC 6913 (Morgan and Harris 1956): A color-magnitude diagram is provided by Hoag *et al.* (1961). HD 194280 (B0 Ib) and KY Cyg (M3.5 Ia) are too distant to be members.

NGC 7235 (Hoag and Applequist 1965): A bright, heavily reddened star (listed also by Becker 1965) has an O or early B spectral type on the basis of its photometric colors and Johnson's Q-value method of determining the unreddened colors; it is not a member. HDE 239895 (B8 Ia) seems too distant to be a member.

b) *Helium Burning*

New evolutionary tracks covering the phase of core helium burning are listed in table 2, which forms a supplement to table 5 of Paper I. Additional theoretical tracks that were terminated before the end of core helium burning include tracks for models based on homogeneous mixing of the intermediate zone: 25 $\mathfrak{M}_\odot$ (Kippenhahn 1969, 1971), 30 $\mathfrak{M}_\odot$ (Barbaro *et al.* 1969), and 50 $\mathfrak{M}_\odot$ (Morris 1970); and tracks for models based on inhomogeneous mixing (or no mixing) of the intermediate zone: 15 $\mathfrak{M}_\odot$ (Simpson 1971) and 9 $\mathfrak{M}_\odot$ and 30 $\mathfrak{M}_\odot$ (Bisnovatyi-Kogan and Nadezhin 1972). The discrepancy noted by Eggen (1971) and by Ferrari *et al.* (1970*b*) between the observed and the (earlier) predicted effective temperatures for red supergiants appears to have been removed by the results of the new models (see also Stothers 1972*a*). Although the new tracks substantiate our earlier conclusions about the location of core helium-burning models in the H-R diagram as a function of various model parameters (Stothers and Chin 1968; Stothers 1969*a*, 1970*c*, 1972*b*), it is unfortunately still unknown theoretically whether helium burning begins or ends (or both) in the red-supergiant configuration, so that the true fractional lifetime τ_{red}/τ is still undetermined.

c) *Carbon Burning and Later Phases*

Late phases of stellar evolution, beginning with core carbon burning, have recently been calculated for 12, 15, 30, and 60 $\mathfrak{M}_\odot$ with and without the inclusion of neutrino emission (Stothers and Chin 1969; Sugimoto 1970*a*, *b;* see also Ikeuchi *et al.* 1972). The evolution is apparently not complicated by thermal instability in the helium-burning shell (Dennis 1971), but the $^{12}C + {}^{12}C$ reaction rate is unfortunately still uncertain (Arnett 1970; Mazarakis and Stephens 1972), though not by so large a factor as was previously believed. Our adoption of the Arnett-Truran (1969) $^{12}C + {}^{12}C$ rate will not significantly bias the (rather insensitive) stellar lifetimes *without* neutrino emission (Stothers and Chin 1969), but the stellar lifetimes *with* neutrino emission will of course be affected. However, the latter lifetimes are expected to be so short that their exact values are unimportant here. The status of $^{12}C + {}^{16}O$, $^{20}Ne + \gamma$, and $^{16}O + {}^{16}O$ is the same as before (Fowler and Hoyle 1964; Reeves 1965, 1966; Hansen and Zaidins 1971).

d) *Mass Loss*

Mass loss, if extensive enough, can relocate a red supergiant on the blue side of the H-R diagram by removing the star's thick convective envelope. Bisnovatyi-Kogan and

TABLE 2

RECENT THEORETICAL EVOLUTIONARY SEQUENCES OF MODELS FOR MASSIVE STARS DURING CORE HELIUM BURNING*

$\mathfrak{M}/\mathfrak{M}_\odot$	X_e	Z_e	Composition in Intermediate Zone	$\tau(10^6$ yr)	τ_{red}/τ	Author
10.....	0.70	0.03	Inhomogeneous	3.4	0.5	Paczyński (1970*a*, *b*)
15.....	0.70	0.03	Inhomogeneous	1.4	1.0	Paczyński (1970*a*, *b*)
15.....	0.60	0.04	Inhomogeneous	...	0.3	Robertson (1971)
15.....	0.60	0.04	Homogeneous	...	0.3	Robertson (1971)
15.....	0.75	0.02	Homogeneous	1.4	0.3	Simpson (1971)
20.....	0.60	0.04	Homogeneous	0.75	0.1	Chiosi and Summa (1970)
20.....	0.60	0.04	Inhomogeneous	0.84	0.2	Chiosi and Summa (1970)
30.....	0.60	0.04	Homogeneous	0.53	0.1	Chiosi and Summa (1970)
30.....	0.75	0.02	Homogeneous	0.55	0.1	Simpson (1971)
30.....	0.75	0.02	Inhomogeneous	0.53	1.0	Simpson (1971)

* The full opacity was used in all sequences.

Nadezhin (1972) have recently computed evolutionary tracks for 9 $\mathfrak{M}_\odot$ and 30 $\mathfrak{M}_\odot$ up to the onset of core helium burning, and have found that, when their model of 30 $\mathfrak{M}_\odot$ crosses the Hertzsprung gap and begins to evolve into a G or K supergiant, the outer envelope develops a density inversion, caused by an excess of the radiative luminosity over the "critical" luminosity, whereupon matter flows off the surface at the heavy rate of about 0.5 $\mathfrak{M}_\odot$ year^{-1}. Their model of 9 $\mathfrak{M}_\odot$ does not encounter this instability, and Bisnovatyi-Kogan and Nadezhin have estimated that the critical mass for extensive outflow is $\sim$20 $\mathfrak{M}_\odot$.

These results are questionable for two major reasons. First of all, development of a density inversion in a nearly radiative envelope may simply lead to stronger convective motions (Spiegel 1971) rather than to such enormously energetic mass outflow if the outflow is to be driven by radiation pressure (cf. Lucy and Solomon 1970). In either event, the envelope (of a massive star) is known to remain dynamically stable against the "ionization mechanism" of mass loss (Paczyński and Ziółkowski 1968; Stothers 1972*a*). Second, and more importantly, very massive M-type supergiants, though rare, do exist. They occur in too large numbers and in too characteristic a position on the H-R diagram to be pre–main-sequence stars (Paper I; Stothers 1972*b*). Their average "pulsational" mass is $\sim$25 $\mathfrak{M}_\odot$ (luminosity class Ia), which agrees well with their "evolutionary" masses based on their luminosities and with their "expected" masses based on the spectral type of the main-sequence turnup in the very young associations to which they belong (Stothers and Leung 1971). The largest masses determined from binary orbits are $\sim$19 $\mathfrak{M}_\odot$ for the K component of 32 Cyg (Wright 1970), $\sim$18 $\mathfrak{M}_\odot$ for the M component of VV Cep (Hutchings and Wright 1971), and $>$34 $\mathfrak{M}_\odot$ for the M component of Boss 1985 (Cowley 1965).

Finally, the *observed* rates of mass loss for M supergiants (most recently those of Gehrz and Woolf 1971) are quite ineffective from the point of view of reducing the stellar masses significantly (Stothers and Chin 1970). As a possible check on these rates, the semiempirical formula of Ezer and Cameron (1971), fitted to the observed rate of mass loss for the Sun, predicts rates for M supergiants that are very similar to those actually observed (Weymann 1962; Woolf 1963; Gehrz and Woolf 1971), although the earlier observational rates of Deutsch (1956) and Wilson (1960) are considerably smaller than the predicted rates.

We conclude that mass loss of the kinds envisaged above is not a likely way of explaining the small number of very massive red supergiants (see § IV).

e) *Rotation*

An evolutionary sequence for 9 $\mathfrak{M}_\odot$ explicitly including (fast) rotation has been calculated by Kippenhahn, Meyer-Hofmeister, and Thomas (1970). During core helium burning the lifetime and track on the H-R diagram do not deviate very much from the case without rotation. During core carbon burning, rotation is found to be even less likely to affect the star's location on the H-R diagram (Sugimoto *et al.* 1968; Stothers and Chin 1969, 1970). These results depend on the simplifying assumption of uniform rotation on the initial upper main sequence, which, nevertheless, has been demonstrated observationally to be approximately correct (Stothers 1972*b*).

f) *Duplicity*

If the primary star in a close binary system expands its radius so as to fill its Roche lobe, then mass transfer from the primary to the secondary is expected to take place, leaving behind a hydrogen-poor remnant and a blue companion of increased mass (Morton 1960). The effect of binary companions on the expected ratio of the numbers of blue and red supergiants is to enhance the number of blue supergiants at the expense of the red supergiants, since many blue supergiants that are in close binary systems will not be able to attain the dimensions of a red supergiant (Paper I; Stothers and Lloyd Evans

1970; Chiosi and Summa 1970). It is now possible to improve considerably on the earlier numerical estimates of this effect.

Three general cases of mass exchange in close binary systems must be distinguished here. In case A the primary's Roche lobe is filled during core hydrogen burning, while in case B it is filled during the envelope-expansion phase immediately following the exhaustion of core hydrogen, and in case BC it is filled either at a later stage of the envelope-expansion phase or at some stage during core helium burning (but before the star becomes a red supergiant).

Case A (Kippenhahn and Weigert 1967; Plavec *et al.* 1968; Horn, Kříž, and Plavec 1970) is found to produce a relatively faint remnant, which, if it subsequently becomes a supergiant, is certainly distinguishable on the H-R diagram from the *bright* supergiant to be expected under normal evolution.

Case B (Kippenhahn and Weigert 1967; Paczyński 1967*a*, *b;* Barbaro *et al.* 1969; Kippenhahn 1969) produces a remnant having a luminosity not much lower than its luminosity before the mass exchange (because its helium core is already fully formed); however, it lies on, or to the left of, the main sequence while burning helium in its core. Although the luminosity class of the remnant might appear to be that of a supergiant (due to its diminished mass-to-luminosity ratio), its unusual location on the H-R diagram would not cause it to be confused with a normal supergiant.

Case BC (Barbaro *et al.* 1969) produces a remnant like that of case B but shifted to a cooler effective temperature; thus a yellow supergiant would yield a remnant that resembles a normal blue supergiant except for an enhancement of certain abundance ratios like He/H and N/C caused by the exposure of layers containing the former products of core hydrogen burning.

In all three cases, if the remnant reexpands its radius during, or at the end of, core helium burning and fills its Roche lobe again, it may lose the remainder of its hydrogen envelope. But the amount of mass that is available to be lost is relatively small, and the remnant will continue to appear during core carbon burning and later phases as a hot, luminous, hydrogen-poor star (possibly as a normal-looking blue supergiant, except for its chemical composition).

The foregoing considerations permit us now to evaluate three extreme possibilities.

1. All the remnants leave the region of supergiants altogether (unrealistic but assumed in Paper I).
2. Only those remnants arising from yellow supergiants appear as more or less normally bright blue supergiants during their further evolution (realistic).
3. All the remnants are indistinguishable from normally bright blue supergiants during their further evolution (unrealistic but assumed, in part, by Chiosi and Summa).

We must consider also three extreme possible schemes of evolution in massive *single* stars during core helium burning. (Core carbon burning and later phases of evolution take place in the red-supergiant configuration among single stars.)

a) Helium burning begins and ends in the blue-supergiant configuration.

b) Helium ignition occurs in the red-supergiant configuration, but helium depletion ensues in the blue-supergiant configuration.

c) Helium burning begins and ends in the red-supergiant configuration.

Theoretical lifetimes have been listed in Paper I for the burning of core helium, carbon, neon, and oxygen. In this paper, we shall include the burning of silicon into iron, since Urca neutrino losses during the last burning phase are no longer expected to be important in accelerating the evolution. The lifetime of silicon burning can be derived from the stellar models of Stothers and Chin (1969) by adopting an energy release of 3×10^{17} ergs g^{-1}.

Observational data are available to estimate what percentage of main-sequence stars can attain the dimensions of a blue supergiant and of a red supergiant, respectively. The most careful discussion of the frequency of (close) spectroscopic binaries among all O and B main-sequence stars, f_{MS}, yields a value of 50 percent, which is independent of spectral class (Jaschek and Gomez 1970). The average frequency observed among the members of nearby associations is likewise independent of spectral class and is 25 percent (Blaauw 1961; van Albada 1968), which becomes about 50 percent after making two corrections to include all mass ratios and all orbital inclinations (van den Heuvel 1969). Individual associations, however, can show frequencies ranging considerably from the quoted average.

The fractions of main-sequence spectroscopic binaries which are able to attain directly blue-supergiant and red-supergiant dimensions, f_b and f_r, can be estimated from the observed distribution of orbital periods among the main-sequence binaries; fortunately, these two fractions are rather insensitive to the other, usually unknown, orbital elements (Jaschek and Jaschek 1965). An examination of various estimates suggests that $f_b =$ 0.3–0.4 and $f_r \leq 0.02$ (Jaschek and Jaschek 1965; van den Heuvel 1969; Chiosi and Summa 1970; Stothers and Lloyd Evans 1970). Because of a selection effect, the true fractions in both cases are probably somewhat higher, and an upper limit for f_r, at least, can be obtained by using (1) 50 percent as the observed frequency of spectroscopic binaries among O and B main-sequence stars and (2) 12 percent as the observed frequency of spectroscopic binaries among M supergiants (Stothers and Lloyd Evans 1970). These figures then imply $f_r = 0.14$, which is an upper limit because main-sequence spectroscopic binaries with the wide separation necessary for the eventual accommodation of a red supergiant have been probably incompletely detected, whereas binary systems containing a red supergiant and a hot companion are easily detected by the appearance of two very different spectra (or by their anomalous colors). However, in order not to underestimate the predicted blue-to-red ratio for supergiants, we shall conservatively adopt $f_{MS} = 0.5$, $f_b = 0.5$, and $f_r = 0$.

With the help of an H-R diagram, it is possible to identify, in clusters and associations, binary remnants that look like *faint supergiants*.[3] Therefore, only the realistic *second* possibility listed above concerning the appearance of the remnants need concern us here. Combining all our results, we arrive at the predicted blue-to-red ratios for supergiants listed in table 3. They are given for the two cases of evolution with and without neutrino emission (ν) as well as for the two cases where close binary systems (CBS) are included and neglected. From this table we may conclude that, for evolved supergiants whose location on the H-R diagram is known, the blue-to-red ratio neglecting neutrino emission is not expected to exceed $\sim$3 *even when the maximum influence of binary companions is taken into account and even though the fraction of a single star's lifetime that is spent burning core helium as a blue supergiant is unknown.*

TABLE 3

PREDICTED UPPER LIMIT ON THE RATIO OF THE NUMBERS OF EVOLVED BLUE AND RED SUPERGIANTS

Single-Star Evolution (case)	15 $\mathfrak{M}_\odot$				30 $\mathfrak{M}_\odot$			
	No ν, No CBS	No ν, CBS	ν, No CBS	ν, CBS	No ν, No CBS	No ν, CBS	ν, No CBS	ν, CBS
(*a*).........	1.9	3.3	$\sim$120	$\sim$180	1.4	2.6	$\sim$110	$\sim$170
(*b*)..........	1.6	2.9	$\sim$ 14	$\sim$ 21	1.3	2.4	$\sim$ 21	$\sim$ 32
(*c*)..........	$\sim$0	0.5	$\sim$ 0	0.5	$\sim$0	0.5	$\sim$ 0	0.5

[3] Remnants of this kind may also include certain of the Wolf-Rayet stars, helium stars, shell stars, and O-type stragglers. Several examples of remnants among the blue supergiants have been pointed out by Stothers and Lloyd Evans (1970), but many of the faintest remnants are expected to be invisible because of the brightness of their more massive main-sequence companions.

IV. OBSERVATIONAL RATIO OF THE NUMBERS OF BLUE AND RED SUPERGIANTS

a) The Milky Way and External Galaxies

The blue-to-red ratio for supergiants in the Galaxy, irrespective of their membership in a stellar group or of their evolutionary status, has been determined by Schild (1970) to be $n_b/n_r = 1.6$ near the Sun; we have confirmed that $n_b/n_r < 2.3$, by modifying Lee's (1970) statistics to include only supergiants among the O and B stars. All known supergiant members of clusters and associations taken together (Humphreys 1970*c*) yield $n_b/n_r = 4$, although the more reliable supergiant data of this paper yield $n_b/n_r = 5$. Thus, there emerges a clear dependence of the blue-to-red ratio on membership in a stellar group. This dependence is understandable because group members tend to be somewhat younger than field stars (Paper I) and younger supergiants show a larger blue-to-red ratio (§ IV*c*).

Walker (1964, 1967) has studied the galactic radial distribution of n_b/n_r for the brightest stars in the spiral galaxy M33 and has found that n_b/n_r increases steeply toward the galactic center. Although Hartwick (1970) used admittedly biased supergiant statistics for distant associations in our own Galaxy (Humphreys 1970*c*)—in the sense that the red supergiants in the southern hemisphere, i.e., toward the galactic center, are known to be very incompletely observed—he found tentative evidence for a trend of n_b/n_r like that in M33. However, n_b/n_r for the brightest stars in the Magellanic Clouds—small, irregular systems—seems to be independent of galactocentric distance (Walker, Blanco, and Kunkel 1969).

Van den Bergh (1968) has attempted to explain the observed effect in M33 by postulating a dependence of supergiant luminosity on metals abundance, which is assumed to vary with galactocentric distance. Calculation of the evolution of massive supergiants, however, shows relatively little dependence of this kind, although a variation in chemical composition can change the blue-to-red ratio via the *effective temperature* (see table 2). However, more useful insight into the problem is provided by table 4, where the blue-to-red ratio is shown as a function of galactocentric distance for only the sample of well-studied clusters and subgroups of associations used in this paper. The tabulated run for our Galaxy is found to be similar to that determined by Walker for M33. By examination of the individual clusters and subgroups used, we find that extremely young stellar groups containing the most luminous (i.e., most massive) supergiants are *relatively less common* farther from the galactic center. Undoubtedly related to this fact is the complete absence of Wolf-Rayet stars, and the paucity of other indicators of youth, in the galactic anticenter direction (e.g., Mikulašek 1969).

b) The h and χ Persei Association

A history of the controversy surrounding this association has been given in Paper I, in which it was pointed out that considerable caution must be exercised in using the

TABLE 4

GALACTIC RADIAL VARIATION OF THE RATIO OF THE NUMBERS OF BLUE AND RED SUPERGIANTS BELONGING TO OPEN CLUSTERS AND SUBGROUPS OF ASSOCIATIONS

R (kpc)	n_b	n_r	n_b/n_r
7–9	18	2	9.0
9–11	18	4	4.5
11–13	10	3	3.3

Perseus supergiants in testing theories of weak interactions. Thus, Bandyopadhyay's (1971) test of his theory of photon-neutrino coupling can be rejected on rather simple grounds (Stothers 1970*b*, 1971*a*), while Simpson's (1971) rediscussion of the current-current theory adds little to the analysis already published in Paper I and by Schild (1970).

A more quantitative analysis of this complex association is now possible. Only the supergiants in the cluster nuclei and in the inner part of the association will be used, on account of the difficulty of segregating background supergiants in the Perseus arm from the outer part of the association (Schild 1967; Bronnikova 1968; Stothers 1969*b*; Humphreys 1970*a*). Although newly remeasured proper motions of the Perseus supergiants cast doubt on the membership of certain blue supergiants presently ascribed to the inner group (Bronnikova 1968), membership of these supergiants is strongly indicated by previously measured proper motions, radial velocities, spectroscopic parallaxes, and positions in the sky. Observational data for the supergiants will be reduced by following the procedure used for the supergiants in the cluster nuclei (Stothers 1972*b*); a list of the supergiant members of I Persei is given in Paper I. The following new results are obtained.

1. The faintest blue and red supergiants have approximately the same luminosity, $M_{\rm bol} \approx -6.1$. This luminosity corresponds to a mass of 10 $\mathfrak{M}_\odot$ (Stothers 1972*b*), which agrees excellently with the "expected" mass of a supergiant evolved from the oldest part of the broad main-sequence turnup (spectral type B2 in the inner group according to Schild 1967).

2. The masses of SU, AD, BU, and FZ Per inferred from their luminosities are in very good agreement with their "pulsational" masses determined from their periods of light variability (Stothers and Leung 1971).

3. The observed red supergiants seem to belong with the faint group of evolved blue supergiants having spectral types B2 and B8–A2. The blue-to-red ratio for these supergiants is 7/9.

4. No red supergiants seem to belong with the very bright group of evolved blue supergiants having early spectral types, B1–B3. The brightest observed blue supergiant has a mass of 22 $\mathfrak{M}_\odot$ (as estimated from its luminosity), in good agreement with the "expected" mass based on the youngest part of the main-sequence turnup (spectral type B0.5 according to Schild). The blue-to-red ratio for these very bright supergiants is 4/0.

Our results are in disagreement with the surmise of Ferrari *et al.* (1970*a*) that many of the red supergiants (including the variables) have masses of 7–9 $\mathfrak{M}_\odot$. Their suggestion was based partly on the inclusion of stars from the heterogeneous outer group of I Persei in the H-R diagram of the association. In fact, their suggestion is not really new, being clearly implicit in the similar H-R diagrams showing theoretical evolutionary tracks as presented by Eggen (1965) and by Schild (1967).

The properties listed above for the supergiant populations belonging to the nuclei and inner group of I Persei seem to accord well with data for other stellar groups (Schild 1970; Stothers 1972*b*), and are particularly reminiscent of the properties of I Geminorum (Paper I) and I Aurigae (Stothers 1971*b*). Furthermore, the blue-to-red ratios appear to be normal, at least within the statistical uncertainty (§ IV*c*; compare also the similar blue-to-red ratios for cluster members in the Large Magellanic Cloud, given in table 8 of Paper I). Although the supergiants in the outer group of I Persei (Paper I) and in other large associations (Humphreys 1970*c*) seem to follow the general pattern of supergiants in the inner group, a trustworthy dependence of the blue-to-red ratio on stellar mass cannot be obtained from members of such complex associations without much further work.

c) Open Clusters and Subgroups of Associations

The reasons for adopting the supergiants of only the well-studied clusters and subgroups of associations listed in this paper and in Paper I have already been made clear.

All 25 of the stellar groups selected for our purpose (given in table 8 of Stothers 1972*b*) are sufficiently evolved to contain, potentially, both blue and red supergiants, since they have main-sequence turnups in the spectral range O7–B2 (with the median spectral type occurring at B0.5). The supergiant membership of these groups is believed to be complete.

The definitely evolved supergiants may be arranged in order of their masses (or ages), in two ways. The first way is to use the spectral type of the main-sequence turnup of the groups to which they belong, and is therefore *independent of distance;* uncertainties associated with the relative luminosities of supergiants in different stellar groups are thus avoided. The corresponding blue-to-red ratios for evolved supergiants are shown in table 5. The second way of arranging the supergiants is to use their luminosities. Transformation from luminosity to mass (or to age) depends somewhat on the adopted phase of evolution for the red supergiants. The corresponding blue-to-red ratios in table 6 are given separately for two different assumptions, namely, that the red supergiants are burning (1) core helium and (2) core carbon. Tables 5 and 6 show that the observed blue-to-red ratio increases steeply with increasing mass (or decreasing age), even though this ratio is still a lower limit (but not to the extent of Paper I) due to our rejection of blue supergiants classed only as "possible" post-main-sequence stars (Stothers 1972*b*).

Comparison of the observed ratios with the predicted ratios of table 3 suggests the following conclusions (which are independent of our theoretical ignorance concerning what side of the H-R diagram a star occupies during core helium burning). First, the large ratio observed at the highest masses indicates that most of the evolution during core helium burning at these masses actually takes place in the blue-supergiant configuration and, further, that a rapid acceleration of evolution must take place after core

TABLE 5

NUMBERS OF EVOLVED BLUE AND RED SUPERGIANTS IN OPEN CLUSTERS AND SUBGROUPS OF ASSOCIATIONS ARRANGED IN ORDER OF THE SPECTRAL TYPE OF THE MAIN-SEQUENCE TURNUP

Sp.	$\mathfrak{M}/\mathfrak{M}_\odot$	n_b (Blue)	n_r (Red)
O5–O9.7.....	25–100	5	0
B0–B0.5.....	15–25	6	1:
B0.7–B1.2...	11–15	7	5
B1.5–B2.....	9–11	4	3

TABLE 6

NUMBERS OF EVOLVED BLUE AND RED SUPERGIANTS IN OPEN CLUSTERS AND SUBGROUPS OF ASSOCIATIONS ARRANGED IN ORDER OF THE LUMINOSITY OF THE SUPERGIANTS

$M_{\rm bol}$	$\mathfrak{M}/\mathfrak{M}_\odot$ (from $M_{\rm bol}$)	n_b (Blue)	n_r (Red, based on He core)	n_r (Red, based on C core)
Very bright...	25–100	6	0	0
Bright.......	20–25	5	0	0
Moderate.....	13–20	7	6	0
Faint........	9–13	4	3	9

helium burning; we ascribe this acceleration to the influence of neutrino emission. Second, if this is so, the small observed ratio for the lower masses must be due to the fact that most of the *observed* red supergiants are burning core helium. There is strong independent evidence for this second conclusion (Stothers and Lloyd Evans 1970; Stothers and Leung 1971; Stothers 1972*b*) based on (*a*) the derived masses of red supergiants and (*b*) the relative luminosities of blue and red supergiants in well-observed clusters.[4]

It is instructive to examine how a blue-to-red ratio of $\sim$11 might be achieved without the assumption of neutrino emission. One method is to assume that the percentage of main-sequence stars of very high mass in close binary systems is at least 90 percent. Observational data for the nearest associations, however, indicate that not only is such a percentage much too high but also the correct percentage is essentially independent of stellar mass in the spectral range O–B5 (§ III*f*).

Another way of obtaining a large blue-to-red ratio is to assume that rapid mass loss transports stars away from the red-supergiant configuration at some stage before core carbon depletion begins. Specifically, Bisnovatyi-Kogan and Nadezhin (1972) have suggested that drastic mass loss occurs for G or K supergiants heavier than $\sim 20\ \mathfrak{M}_\odot$, and that the observable remnants might be Wolf-Rayet stars. (Our data show a ratio of 11 massive evolved blue supergiants to four Wolf-Rayet stars in the youngest clusters.) But the contention of Bisnovatyi-Kogan and Nadezhin is difficult to reconcile with other data for late-type supergiants (§ III*d*) and, more importantly, with the fact that at least three of our four cluster Wolf-Rayet stars are in binary systems with short orbital periods (85, 8.8, and 3.3: days) and therefore could never have been G or K supergiants. Rather, we regard these binary Wolf-Rayet stars as probable remnants of mass exchange (Paczyński 1967*b*) or of pulsational mass loss (Simon and Stothers 1970; Schmidt-Kaler 1970). An alternative suggestion for rapid mass loss might be based on the theoretical result that stars heavier than $\sim 20\ \mathfrak{M}_\odot$ develop very deep convective envelopes, extending down to the carbon-oxygen zone, after the phase of core helium burning (Stothers and Chin 1969); if metal enrichment of the envelope were somehow to lead to a stronger stellar wind or to dynamical instability, major mass loss might take place. But this suggestion seems to be contradicted by the fact that no carbon stars of any color are observed in our stellar groups (§ V).

Finally, one might argue that very thick circumstellar dust clouds are obscuring the more massive M supergiants to the point that these objects have remained mostly undetected (like the hypothetical "black giant" stars of Bondi, Gold, and Hoyle 1955). While it is true that the *visual extinction* is found to be correlated with luminosity for M supergiants (e.g., Gehrz and Woolf 1971; Humphreys *et al.* 1972), the actual discovery of M supergiants has been based mostly on deep-sky *infrared* surveys, covering virtually the whole galactic plane in the northern hemisphere and part of the galactic plane in the southern hemisphere (including the region of I Scorpii), and therefore has been less affected by the presence of dust clouds.

V. CONCLUSION

The main aim of the present paper has been to study the variation of the blue-to-red ratio for supergiants as a function of stellar mass. For this purpose an empirical blue-to-red ratio has been determined which is expected to be particularly useful for evolutionary studies. Most of the recent theoretical calculations of stellar evolution at high mass tend to support the earlier calculations quoted and used in Paper I, and thereby lend confidence to our conclusion that the essential features needed to test for neutrino emission are now sufficiently well known despite our theoretical ignorance of the colors of core-helium-burning models. Enough information on rotation, mass loss, and duplicity is also available so that we can reliably predict limits on the blue-to-red ratio.

[4] The effect shows up indirectly in table 6 if one compares the observed numbers of red supergiants there with the observed numbers in table 5.

Our main conclusions follow.

1. The observed paucity of red supergiants at very high mass confirms and extends the trend observed in Paper I, and thereby supports the conclusion that the carbon-burning and later phases of evolution are very rapid; the only likely accelerating mechanism is neutrino emission. Since carbon probably burns at a higher temperature than previously believed, the pair-annihilation process is now expected to be more important than the photoneutrino process.

2. A close examination of the controversial h and χ Persei association reveals the same dependence of the blue-to-red ratio on stellar mass as is observed for other clusters and subgroups of associations in our Galaxy (and in the Large Magellanic Cloud).

3. The observed decrease of the blue-to-red ratio with increasing galactocentric distance in M33 (Walker 1964, 1967) seems to be mirrored in our own Galaxy (as suggested by our present supergiant data) and is apparently due to a relative scarcity of extremely young aggregates in distant galactic regions.

Finally, we point out the striking absence of carbon and S-type stars in our sample of massive stars. Although one N star is located near NGC 457, its visual absolute magnitude, if it is a member, would be only about -1.4 (Gordon 1968).[5] This observed paucity of carbon and S-type stars among the supergiants of very high mass, as interpreted in our previous paper (Stothers and Chin 1969), may be considered further evidence for neutrino emission.

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[5] N and S stars that are possible members of other young clusters have been listed in Paper I. To these can be added two candidates of N type near NGC 6883, a cluster which apparently contains no supergiants (Purgathofer 1961; Gordon 1968). Most of these objects, however, appear to be somewhat older than the youngest O and B stars (Westerlund 1964; Gordon 1968; Peery 1971).

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